Mixing and CP-violation in charm

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The motivation most often cited in searches for $D^0 - \overline{D^0}$ mixing and CP-violation in charm system lies with the possibility of observing a signal from new physics which dominates that from the Standard Model. We review recent theoretical predictions and experimental constraints on $D^0 - \overline{D^0}$ mixing parameters. We also discuss the current status of searches for CP-violation in charmed meson transitions, as well as some recent theoretical ideas.

1. Introduction

Charm transitions play a unique dual role in the modern investigations of flavor physics. It provides valuable supporting measurements for studies of CP-violation in B-decays, such as form-factors and decays constants, as well as outstanding opportunities for indirect searches for physics beyond the Standard Model (SM). It must be noted that in many dynamical models of new physics the effects of new particles observed in s, c, and b transitions are correlated. Therefore, such combined studies could yield the most stringent constraints on their parameters. For example, loop-dominated processes such as $D^0 - \overline{D^0}$ mixing or flavor-changing neutral current (FCNC) decays are influenced by the dynamical effects of *down-type particles* [1], whereas up-type particles are responsible for FCNC in the beauty and strange systems. Finally, from the practical point of view, charm physics experiments provide outstanding opportunities for studies of new physics because of the availability of large statistical samples of data.

The basic idea behind searches for new physics in the relatively low energy processes like charm transitions stems from the fact that the effects of new interactions in can be naturally written in terms of a series of local operators of increasing

dimension. Together with the one loop Standard Model effects [2] they can contribute to $\Delta C = 1$ (decays) or $\Delta C = 2$ (mixing) FCNC transitions. For example, in the case of $D^0 - \overline{D^0}$ mixing these operators generate contributions to the effective operators that change D^0 state into $\overline{D^0}$ state, leading to the mass eigenstates

$$|D_{\frac{1}{2}}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle, \qquad (1)$$

where the complex parameters p and q are obtained from diagonalizing the $D^0 - \overline{D^0}$ mass matrix. Note that $|p|^2 + |q|^2 = 1$. If CP-violation in mixing is neglected, p becomes equal to q, so $|D_{1,2}\rangle$ become CP eigenstates, $CP|D_\pm\rangle = \pm|D_\pm\rangle$. The mass and width splittings between these eigenstates are given by

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}. \qquad (2)$$

It is known experimentally that $D^0 - \overline{D^0}$ mixing proceeds extremely slowly, which in the Standard Model is usually attributed to the absence of superheavy quarks destroying GIM cancellations. This situation is an exact opposite to what happens in the B system, where $B^0 - \overline{B^0}$ mixing measurements are used to constrain top quark couplings [3].

It is instructive to see how new physics can affect charm mixing. Since the lifetime difference y is constructed from the decays of D into physical states, it should be dominated by the Standard Model contributions, unless new physics significantly modifies $\Delta C = 1$ interactions. On the

contrary, the mass difference x can receive contributions from all energy scales. Thus, it is usually conjectured that new physics can significantly modify x leading to the inequality $x \gg y$ [2].

Another possible manifestation of new physics interactions in the charm system is associated with the observation of (large) CP-violation. This is due to the fact that all quarks that build up the hadronic states in weak decays of charm mesons belong to the first two generations. Since 2×2 Cabbibo quark mixing matrix is real, no CP-violation is possible in the dominant tree-level diagrams which describe the decay amplitudes. CP-violating amplitudes can be introduced in the Standard Model by including penguin or box operators induced by virtual b-quarks. However, their contributions are strongly suppressed by the small combination of CKM matrix elements $V_{cb}V_{ub}^*$. It is thus widely believed that the observation of (large) CP violation in charm decays or mixing would be an unambiguous sign for new physics. This fact makes charm decays a valuable tool in searching for new physics, since the statistics available in charm physics experiment is usually quite large.

As in B-physics, CP-violating contributions in charm can be generally classified by three different categories: (I) CP violation in the decay amplitudes. This type of CP violation occurs when the absolute value of the decay amplitude for D to decay to a final state f (A_f) is different from the one of corresponding CP-conjugated amplitude ("direct CP-violation"); (II) CP violation in $D^0 - \overline{D^0}$ mixing matrix. This type of CP violation is manifest when $R_m^2 = |p/q|^2 = (2M_{12} - i\Gamma_{12})/(2M_{12}^* - i\Gamma_{12}^*) \neq 1$; and (III) CP violation in the interference of decays with and without mixing. This type of CP violation is possible for a subset of final states to which both D^0 and $\overline{D^0}$ can decay.

For a given final state f, CP violating contributions can be summarized in the parameter

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} = R_m e^{i(\phi+\delta)}\left|\frac{\overline{A}_f}{A_f}\right|, \qquad (3)$$

where A_f and $\overline{A}_f$ are the amplitudes for $D^0 \to f$

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[2]This signal for new physics is lost if a relatively large y, of the order of a percent, is observed [4].

and $\overline{D^0} \to f$ transitions respectively and δ is the strong phase difference between A_f and $\overline{A}_f$. Here ϕ represents the convention-independent weak phase difference between the ratio of decay amplitudes and the mixing matrix. Since CP-violation in the mixing matrix is expected to be small, we will often expand $R_m^{\pm 2} = 1 \pm A_m$ [4].

At present, most experimental information about the $D^0 - \overline{D^0}$ mixing parameters x and y comes from the time-dependent analyses that can roughly be divided into two categories. First, more traditional studies look at the time dependence of $D \to f$ decays, where f is the final state that can be used to tag the flavor of the decayed meson. The most popular is the non-leptonic doubly Cabibbo suppressed (DCS) decay $D^0 \to K^+\pi^-$. Time-dependent studies allow one to separate the contribution of direct doubly Cabibbo suppressed transition from the one involving mixing $D^0 \to \overline{D^0} \to K^+\pi^-$,

$$\Gamma[D^0 \to K^+\pi^-] = e^{-\Gamma t}|A_{K^-\pi^+}|^2$$
$$\times \left[R + \sqrt{R}R_m(y'\cos\phi - x'\sin\phi)\Gamma t \qquad (4) \right.$$
$$\left. + \frac{R_m^2}{4}(y^2 + x^2)(\Gamma t)^2 \right],$$

where $\sqrt{R}e^{i\delta}$ parameterizes the ratio of DCS and Cabibbo favored (CF) decay amplitudes, and $x' = x\cos\delta + y\sin\delta$, $y' = y\cos\delta - x\sin\delta$. Since x and y are small, the best constraint comes from the linear terms in t that are also *linear* in x and y. As follows from Eq. (4) direct extraction of x and y is not possible due to unknown relative strong phase δ of DCS and CF amplitudes [5]. This phase, however, can be measured in an experiment which produces CP-correlated pairs of D-mesons, such as CLEO-c [6]. Recent experimental constraints on $D^0 - \overline{D^0}$ mixing parameters from $D^0 \to K^+\pi^-$ analyses are presented in Table 1 (presented Belle and FOCUS results assume conservation of CP).

Second, D^0 mixing parameters can be measured by comparing the lifetimes extracted from the analysis of D decays into the CP-even and CP-odd final states. This study is also sensitive

Table 1
Recent measurements of x and y in $D^0 \to K^+\pi^-$

Experiment	x'^2 (95% CL) $(\times 10^{-3})$	y' (95% CL) $(\times 10^{-3})$
Belle (2004)	< 0.81	$-8.2 < y' < 16$
BaBar (2003)	< 2.2	$-56 < y' < 39$
FOCUS (2001)	< 1.52	$-124 < y' < -5$
CLEO (2000)	< 0.82	$-58 < y' < 10$

The experimental values are given in ref. [7].

to a *linear* function of y via

$$\frac{\tau(D \to K^-\pi^+)}{\tau(D \to K^+K^-)} - 1 = y\cos\phi - x\sin\phi\frac{A_m}{2}. \quad (5)$$

The results from various experiments can be found in ref. [8], with the grand average $y_{CP} = (0.9 \pm 0.4)\%$.

Time-integrated studies of the semileptonic transitions are sensitive to the *quadratic* form $x^2 + y^2$ and at the moment are not competitive with the analyses discussed above.

The construction of tau-charm factories CLEO-c and BES-III allows for new *time-independent* methods that are sensitive to a linear function of y. One can use the fact that heavy meson pairs produced in the decays of heavy quarkonium resonances have the useful property that the two mesons are in the CP-correlated states [9]. For instance, by tagging one of the mesons as a CP eigenstate, a lifetime difference may be determined by measuring the leptonic branching ratio of the other meson. Its semileptonic *width* should be independent of the CP quantum number since it is flavor specific, yet its *branching ratio* will be inversely proportional to the total width of that meson. Since we know whether this $D(k_2)$ state is tagged as a (CP-eigenstate) $D_\pm$ from the decay of $D(k_1)$ to a final state S_σ of definite CP-parity $\sigma = \pm$, we can easily determine y in terms of the semileptonic branching ratios of $D_\pm$, which we denote $\mathcal{B}^\ell_\pm$. Neglecting small CP-violating effects,

$$y = \frac{1}{4}\left(\frac{\mathcal{B}^\ell_+(\mathcal{D})}{\mathcal{B}^\ell_-(\mathcal{D})} - \frac{\mathcal{B}^\ell_-(\mathcal{D})}{\mathcal{B}^\ell_+(\mathcal{D})}\right). \quad (6)$$

A more sophisticated version of this formula as

well as studies of feasibility of this method can be found in ref. [9].

The current experimental upper bounds on x and y are on the order of a few times 10^{-2}, and are expected to improve significantly in the coming years. To regard a future discovery of nonzero x or y as a signal for new physics, we would need high confidence that the Standard Model predictions lie well below the present limits. As was recently shown [10], in the Standard Model, x and y are generated only at second order in $SU(3)_F$ breaking,

$$x\,,\,y \sim \sin^2\theta_C \times [SU(3)\ \text{breaking}]^2\,, \quad (7)$$

where θ_C is the Cabibbo angle. Therefore, predicting the Standard Model values of x and y depends crucially on estimating the size of $SU(3)_F$ breaking. Although y is expected to be determined by the Standard Model processes, its value nevertheless affects significantly the sensitivity to new physics of experimental analyses of D mixing [4].

Theoretical predictions of x and y within and beyond the Standard Model span several orders of magnitude [11] (see Fig. 1). Roughly, there are two approaches, neither of which give very reliable results because m_c is in some sense intermediate between heavy and light. The "inclusive" approach is based on the operator product expansion (OPE). In the $m_c \gg \Lambda$ limit, where Λ is a scale characteristic of the strong interactions, ΔM and $\Delta \Gamma$ can be expanded in terms of matrix elements of local operators [12]. Such typically calculations yield $x, y < 10^{-3}$. The use of the OPE relies on local quark-hadron duality, and on Λ/m_c being small enough to allow a truncation
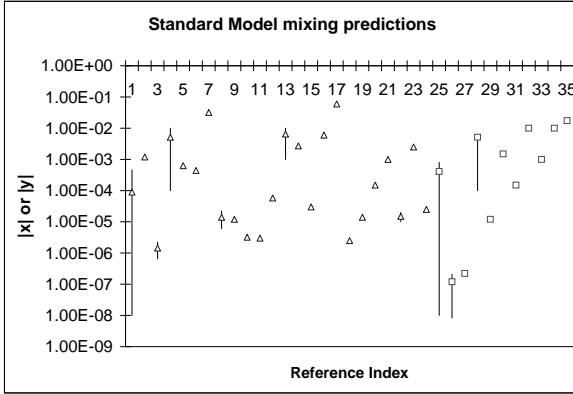
Figure 1. Standard Model predictions for $|x|$ (open triangles) and $|y|$ (open squares). Horizontal line references are tabulated in [11].

of the series after the first few terms. The charm mass may not be large enough for these to be good approximations, especially for nonleptonic D decays. An observation of y of order 10^{-2} could be ascribed to a breakdown of the OPE or of duality, but such a large value of y is certainly not a generic prediction of OPE analyses. The "exclusive" approach sums over intermediate hadronic states, which may be modeled or fit to experimental data [13]. Since there are cancellations between states within a given $SU(3)$ multiplet, one needs to know the contribution of each state with high precision. However, the D is not light enough that its decays are dominated by a few final states. In the absence of sufficiently precise data on many decay rates and on strong phases, one is forced to use some assumptions. While most studies find $x, y < 10^{-3}$, Refs. [13] obtain x and y at the 10^{-2} level by arguing that $SU(3)_F$ violation is of order unity. It was also shown that phase space effects alone provide enough $SU(3)_F$ violation to induce $x, y \sim 10^{-2}$ [10]. Large effects in y appear for decays close to D threshold, where an analytic expansion in $SU(3)_F$ violation is no longer possible; a dispersion relation can then be used to show that x would receive contributions

of similar order of magnitude.

The above discussion shows that, contrary to B and K systems, theoretical calculations of x and y are quite uncertain [14], and the values near the current experimental bounds cannot be ruled out. Therefore, it will be difficult to find a clear indication of physics beyond the Standard Model in $D^0 - \overline{D^0}$ mixing measurements alone. The only robust potential signal of new physics in charm system at this stage is observation of large violation of CP.

CP violation in D decays and mixing can be searched for by a variety of methods. For instance, time-dependent decay widths for $D \rightarrow K\pi$ are sensitive to CP violation in mixing (see Eq.(4)). Provided that the x and y are comparable to experimental sensitivities, a combined analysis of $D \rightarrow K\pi$ and $D \rightarrow KK$ can yield interesting constraints on CP-violating parameters [4].

Most of the techniques that are sensitive to CP violation make use of the decay asymmetry,

$$A_{CP}(f) = \frac{\Gamma(D \rightarrow f) - \Gamma(\overline{D} \rightarrow \overline{f})}{\Gamma(D \rightarrow f) + \Gamma(\overline{D} \rightarrow \overline{f})}. \qquad (8)$$

Most of the properties of Eq. (8), such as dependence on the strong final state phases, are similar to the ones in B-physics [15]. Current experimental bounds from various experiments, all consistent with zero within experimental uncertainties, can be found in [16].

Other interesting signals of CP-violation that are being discussed in connection with tau-charm factory measurements are the ones that are using quantum coherence of the initial state. An example of this type of signal is a decay $(D^0\overline{D^0}) \rightarrow f_1 f_2$ at $\psi(3770)$ with f_1 and f_2 being the different final CP-eigenstates of the same CP-parity. This type of signals are very easy to detect experimentally. The corresponding CP-violating decay rate for the final states f_1 and f_2 is

$$\Gamma_{f_1 f_2} = \frac{1}{2R_m^2} \Big[\left(2 + x^2 - y^2\right) |\lambda_{f_1} - \lambda_{f_2}|^2$$
$$+ \left(x^2 + y^2\right) |1 - \lambda_{f_1}\lambda_{f_2}|^2 \Big] \Gamma_{f_1}\Gamma_{f_2}. \quad (9)$$

The result of Eq. (9) represents a slight generalization of the formula given in Ref. [17]. It is clear

that both terms in the numerator of Eq. (9) receive contributions from CP-violation of the type I and III, while the second term is also sensitive to CP-violation of the type II. Moreover, for a large set of the final states the first term would be additionally suppressed by $\mathrm{SU}(3)_F$ symmetry, as for instance, $\lambda_{\pi\pi} = \lambda_{KK}$ in the $\mathrm{SU}(3)_F$ symmetry limit. This expression is of the *second* order in CP-violating parameters (it is easy to see that in the approximation where only CP violation in the mixing matrix is retained, $\Gamma_{f_1 f_2} \propto \left| 1 - R_m^2 \right|^2 \propto A_m^2$).

The existing experimental constraints [18] demonstrate that CP-violating parameters are quite small in the charm sector, regardless of whether they are produced by the Standard Model mechanisms or by some new physics contributions. Since the above measurements involve CP-violating decay *rates*, these observables are of *second order* in the small CP-violating parameters, a challenging measurement.

It is also easy to see that the rate asymmetries of Eq. (8) require tagging of the initial state with the consequent reduction of the dataset. In that respect, it is important to maximally exploit the available statistics.

It is possible to use a method that both does not require flavor or CP-tagging of the initial state and results in the observable that is *first order* in CP violating parameters [19]. Let's concentrate on the decays of D-mesons to final states that are common for D^0 and $\overline{D^0}$. If the initial state is not tagged the quantities that one can easily measure are the sums

$$\Sigma_i = \Gamma_i(t) + \overline{\Gamma}_i(t) \qquad (10)$$

for $i = f$ and $\overline{f}$. A CP-odd observable which can be formed out of Σ_i is the asymmetry

$$A_{CP}^U(f,t) = \frac{\Sigma_f - \Sigma_{\overline{f}}}{\Sigma_f + \Sigma_{\overline{f}}} \equiv \frac{N(t)}{D(t)}. \qquad (11)$$

We shall consider both time-dependent and time-integrated versions of the asymmetry (11). Note that this asymmetry does not require quantum coherence of the initial state and therefore is accessible in any D-physics experiment. It is expected that the numerator and denominator of

Eq. (11) would have the form,

$$
\begin{aligned}
N(t) &= \Sigma_f - \Sigma_{\overline{f}} = e^{-\mathcal{T}} \left[A + B\mathcal{T} + C\mathcal{T}^2 \right], \\
D(t) &= 2e^{-\mathcal{T}} \left[|A_f|^2 + \left| A_{\overline{f}} \right|^2 \right], \qquad (12)
\end{aligned}
$$

where we neglected direct CP violation in $D(t)$. Integrating the numerator and denominator of Eq. (11) over time yields

$$A_{CP}^U(f) = \frac{1}{D} \left[A + B + 2C \right], \qquad (13)$$

where $D = \Gamma \int_0^\infty dt\, D(t)$.

Both time-dependent and time-integrated asymmetries depend on the same parameters A, B, and C. The result is

$$
\begin{aligned}
A &= |A_f|^2 - \left| \overline{A}_{\overline{f}} \right|^2 - \left| A_{\overline{f}} \right|^2 + \left| \overline{A}_f \right|^2, \\
B &= -2y\sqrt{R} \left[\sin\phi \sin\delta \left(\left| \overline{A}_f \right|^2 + \left| A_{\overline{f}} \right|^2 \right) \right. \\
&\quad \left. - \cos\phi \cos\delta \left(\left| \overline{A}_f \right|^2 - \left| A_{\overline{f}} \right|^2 \right) \right], \qquad (14) \\
C &= \frac{x^2}{2} A.
\end{aligned}
$$

We neglect small corrections of the order of $\mathcal{O}(A_m x, r_f x, ...)$ and higher. It follows that Eq. (14) receives contributions from both direct and indirect CP-violating amplitudes. Those contributions have different time dependence and can be separated either by time-dependent analysis of Eq. (11) or by the "designer" choice of the final state. Note that this asymmetry is manifestly *first* order in CP-violating parameters.

In Eq. (14), non-zero value of the coefficient A is an indication of direct CP violation. This term is important for singly Cabibbo suppressed (SCS) decays. The coefficient B gives a combination of a contribution of CP violation in the interference of the decays with and without mixing (first term) and direct CP violation (second term). Those contributions can be separated by considering DCS decays, such as $D \to K^{(*)}\pi$ or $D \to K^{(*)}\rho$, where direct CP violation is not expected to enter. The coefficient C represents a contribution of CP-violation in the decay amplitudes after mixing. It is negligibly small in the SM and all models of new physics constrained by

the experimental data. Note that the effect of CP-violation in the mixing matrix on A, B, and C is always subleading.

Eq. (14) is completely general and is true for both DCS and SCS transitions. Neglecting direct CP violation we obtain a much simpler expression,

$$A = 0, \qquad C = 0,$$
$$B = -2y \sin\delta \sin\phi \sqrt{R} \left[\left| \overline{A}_f \right|^2 + \left| A_{\overline{f}} \right|^2 \right] \quad (15)$$

For an experimentally interesting DCS decay $D^0 \to K^+\pi^-$ this asymmetry is zero in the flavor $SU(3)_F$ symmetry limit, where $\delta = 0$ [20]. Since $SU(3)_F$ is badly broken in D-decays, large values of $\sin\delta$ [5] are possible. At any rate, regardless of the theoretical estimates, this strong phase could be measured at CLEO-c. It is also easy to obtain the time-integrated asymmetry for $K\pi$. Neglecting small subleading terms of $\mathcal{O}(\lambda^4)$ in both numerator and denominator we obtain

$$A_{CP}^U(K\pi) = -y \sin\delta \sin\phi \sqrt{R}. \quad (16)$$

It is important to note that both time-dependent and time-integrated asymmetries of Eqs. (15) and (16) are independent of predictions of hadronic parameters, as both δ and R are experimentally determined quantities and could be used for model-independent extraction of CP-violating phase ϕ. Assuming $R \sim 0.4\%$ and $\delta \sim 40^o$ [5] and $y \sim 1\%$ one obtains $\left| A_{CP}^U(K\pi) \right| \sim (0.04\%) \sin\phi$. Thus, one possible challenge of the analysis of the asymmetry Eq. (16), is that it involves a difference of two large rates, $\Sigma_{K^+\pi^-}$ and $\Sigma_{K^-\pi^+}$, which should be measured with the sufficient precision to be sensitive to A_{CP}^U, a problem tackled in determinations of tagged asymmetries in $D \to K\pi$ transitions.

Alternatively, one can study SCS modes, where $R \sim 1$, so the resulting asymmetry could be $\mathcal{O}(1\%) \sin\phi$. However, the final states must be chosen such that A_{CP}^U is not trivially zero. For example, decays of D into the final states that are CP-eigenstates would result in zero asymmetry (as $\Gamma_f = \Gamma_{\overline{f}}$ for those final states) while decays to final states like K^+K^{*-} or $\rho^+\pi^-$ would not. It is also likely that this asymmetry is larger

than the estimate given above due to contributions from direct CP-violation (see eq. 14).

The final state f can also be a multiparticle state. In that case, more untagged CP-violating observables could be constructed. For example, untagged studies of Dalitz plot population asymmetries resulting from the enantiometric intermediate states were proposed in [21] to study direct CP-violation in B_d decays. A similar study is possible here as well.

As any rate asymmetry, Eq. (11) requires either a "symmetric" production of D^0 and $\overline{D^0}$, a condition which is automatically satisfied by all $p\overline{p}$ and e^+e^- colliders, or a correction for $D^0/\overline{D^0}$ production asymmetry.

In summary, charm physics, and in particular studies of CP-violation, could provide new and unique opportunities for indirect searches for new physics. Expected large statistical samples of charm data will allow new sensitive measurements of charm mixing and CP-violating parameters.

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